Are All ‘Mistakes’ Irrational?
On Transitivity and Regret

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You need to hire an analyst. Two candidates applied. You ask them:

- **What is the probability that the last digit of the DJ index at the closing of trade on May 1 will be odd? even?**

Candidate 1: Odd: 80%, Even: 20%

Candidate 2: Odd: 49%, Even: 49%

Which of the two will you hire? (There are no other candidates and you need to get one).

I'd hire the second. The first one is an idiot. The second one just a little careless.

Even though the second violates the rule that

- **The probability of the union of two disjoint events should be the sum of their probabilities**

While the first person observes it.

Two candidates. You ask the first what is 8×7 and he says 54.

You ask the second what is 5×7 and he says 53.

Again, you have to hire one of the two. Which one will you choose?

The absolute mistake of the first is 2, and of the second 18.

Yet it is clear that the first one doesn't know the multiplication table (he remembers two correct answers between 50 and 60, but doesn't know which of the two belongs to 7×8 and which to 6×9).

The other candidate's mistake is a typo — 53 instead of 35.

Johnny's mother has four children:

- **North**
- **South**
- **East**
- **And. . . ?**

Johnny's mother has four children:

- **North**
- **South**
- **East**
- **And Johnny (of course, why West?)**

What is common to all these questions is that

- **They involve obvious mistakes**

- **Sujects will correct their mistakes: 49-49 will become 50-50, 53 will become 35, and West will become Johnny.**

But what about violations of transitivity?

Definition: Preferences are transitive if for all a, b, c:

If $a \succ b$ and $b \succ c$, then $a \succ c$.

Violations of transitivity lead to a "Dutch book" — a sequence of transacions that leave the decision maker worse off.

Suppose the decision maker prefers a to b, b to c, but he also prefers c to a.

Suppose he holds a.

Offer him, for a small amount of money ε, to switch to c. He'll agree.

Offer him now to switch from c to b. He'll agree.

Finally, offer him to switch from b to a. Once again, he'll agree.

So he started with a, ended with a, but paid ε for it!

But consider the following question by Fishburn and LaValle. A fair die is thrown. What do you prefer, A or B?

	1	**2**	**3**	**4**	**5**	**6**
A	**100**	**200**	**300**	**400**	**500**	**600**
B	**200**	**300**	**400**	**500**	**600**	**100**

In 5 of 6 cases, B yields a better outcome, so even though ex ante A and B are the same, ex post the decision maker will be happier in these cases if he picks B and not A.

In one case A is (much) better than B. So it is possible to have $B \succ A$ or $A \succ B$.

But then, by the same reasoning, if $B \succ A$, then $C \succ B$, $D \succ C$, $E \succ D$, $F \succ E$, and $A \succ F$, hence a nontransitive cycle.

	1	2	3	4	5	6
A	100	200	300	400	500	600
B	200	300	400	500	600	100
C	300	400	500	600	100	200
D	400	500	600	100	200	300
E	500	600	100	200	300	400
F	600	100	200	300	400	500
A	100	200	300	400	500	600

Loomes, Starmer, Sugden (1991): Given three events S_1, S_2, S_3 with the probabilities $0.3, 0.3, 0.4$), which will you choose in $\{A, B\}$, $\{B, C\}$, and $\{A, C\}$, where

Event: Probability:	S_1 0.3	S_2 0.3	S_3 0.4	Expected Value
A	18	0	0	5.4
B	8	8	0	4.8
C	4	4	4	4

Common answers (about half of the subjects):

$$c(A, B) = B,\ c(B, C) = C,\ c(A, C) = A$$

Possible justification: Regret (Loomes and Sugden, 1982; Bell 1982).

Key idea: The anticipated utility from an outcome depends not only on the outcome but also on the outcome that could have been obtained.

The decision maker feels elation or regret.

He takes the expected value of these feelings ex ante.

n disjoint states of nature $s_1, \ldots, s_n$ with probabilities $p_1, \ldots, p_n$ such that $\sum p_i = 1$.

A regret function $\psi(x, y)$ which is the regret ($x < y$) or elation ($x > y$) the decision maker feels if he won x when the rejected alternative yielded y.

Two gambles: $X = (x_1, s_1; \ldots; x_n, s_n)$ and $Y = (y_1, s_1; \ldots; y_n, s_n)$.

$X \succeq Y$ iff

$$\sum_i p_i \psi(x_i, y_i) > 0$$

Assumptions on ψ:

- **ψ is skew symmetric:**

$$\psi(x, y) = -\psi(y, x)$$

- **In particular, $\psi(x, x) = 0$ for all x.**

- **$\psi(x, y)$ is increasing in x and decreasing in y.**

- **Regret aversion: For $x > y > z$,**

$$\psi(x, z) > \psi(x, y) + \psi(y, z)$$

Regret theory is very convincing (at least, from a psychological point of view), but unless $\psi(x, y) = u(x) - u(y)$, it must lead to violations of transitivity.

But if $\psi(x, y) = u(x) - u(y)$, then

$$X \succeq Y \iff$$

$$\sum_i p_i \psi(x_i, y_i) \geq 0 \iff$$

$$\sum_i p_i \left[u(x_i) - u(y_i)\right] \geq 0 \iff$$

$$\sum_i p_i u(x_i) \geq \sum_i p_i u(y_i)$$

Which is expected utility thoery.

The “problem” is actually much bigger.

Bikhchandani and Segal, 2011: Suppose that $X \succeq Y$ if, and only if

$$V(\psi_1(x_1, y_1), p_1; \ldots; \psi(x_n, y_n), p_n) \geq 0$$

Then unless V is the sum of the regrets and $\psi(x, y) = u(x) - u(y)$, the preferences $\succeq$ must violate transitivity.

In other words, the only way to avoid such cycles is to go back to expected utility.

So what do we do?

On the one hand, the idea of regret is very compelling, and is strongly supported by many experiments.

On the other hand, it must lead to violatios of transitivity, which is a real taboo in economics.

But is transitivity really that important?

The Talmud, after some deliberation, concludes the following:

- **Tempel service is more important than observing the Shabbat.**

- **The Shabbat is more important than the execution of condemned criminals.**

- **The execution of condemned criminals is more important than the tempel service.**

One solution offered by the Talmud to his puzzle is that we shouldn't be bothered by this cycle, as the three criteria never apply simultaneously.

A similar answer may apply to situations involving regret.

The choice of a over b holds when the choice set is $\{a, b\}$ and the choice of b over c holds when the choice set is $\{b, c\}$.

But if the choice set is never $\{a, b, c\}$, then choosing c out of $\{a, c\}$ should not trouble us.

To sum:

- **Mistakes happen.**

- **Sometimes it is not a real problem, as it will be easily corrected.**

- **But there are situations where mistakes happen because decision makers follow rules that seem intuitive and reasonable.**

- **Policy makers should take (only) such "mistakes" into consideration.**