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Pollution Control in an Uncertain Environment

by

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Pollution Control in an Uncertain Environment

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Abstract:

The effects of the threat of occurrence of environmental catastrophes on optimal pollution control are considered. Recent analysis of irreversible events is extended to two types of reversible events: single-occurrence and multiple-occurrence (recurrent) events. While it is possible that the threat of irreversible events induces more pollution, we show that reversible events, under quite general conditions, induce more conservation (less pollution). The analysis is carried out via a simple method (the " $h\delta$ -method") to identify optimal steady states by comparing steady state policies with small variations from them. For recurrent events the optimal state process must approach a unique steady state from any initial level. In this case, the $h\delta$ -method characterizes the dynamic behavior of the optimal state process without actually solving for it.

Key words: Pollution management; Environmental uncertainty; Reversible events

JEL Classification: Q20, Q25, Q28

1. Introduction

Some consequences of increased pollution of air, water and soil occur abruptly or over a short period of time. Such is the case, for instance, with the outbreak of pollution-induced diseases, or the collapse of an ecosystem as one of its links ceases to perform. Avoiding or preparing for such catastrophes is particularly difficult when occurrence conditions involve uncertainty.

The literature on consumption/pollution tradeoff is rich (see e.g. Tsur and Zemel, henceforth referred to as T&Z, 1996a, and references therein). In this work, we focus on event uncertainty of the type introduced by Cropper (1976) and analyzed by Clarke and Reed (1994) (henceforth referred to as C&R). Such events are triggered by stochastic environmental (exogenous) conditions and their occurrence hazard depends only on the current pollution level. In contrast, for the endogenous events considered by T&Z (1994, 1995, 1996a) the occurrence probability depends also on the pollution history as well as on its current trend.

The exogenous events considered by C&R bear another important feature: they are irreversible in the sense that their occurrence stops the process for ever (either because life ceases to exist or because the resource under planning is rendered obsolete). In contrast, when the process can continue following occurrence, the event is reversible and its effect is manifested in terms of a penalty inflicted upon occurrence.

The effect of uncertainty, regardless of its type, is analyzed via a comparison with a situation in which the event cannot occur. For this non-event problem, there exists a unique equilibrium state to which the optimal state process (the

pollutants concentration) converges from any initial state level. The presence of event threat shifts this equilibrium level. We say that uncertainty induces conservation if the corresponding equilibrium state entails less pollution than its non-event counterpart.

C&R find that exogenous, irreversible events often (though not always) lead to less conservation (more pollution), as compared with the non-event situation. This behavior differs markedly from that derived by T&Z (1994, 1995, 1996a) for endogenous events, in which the event occurs as soon as the state reaches a fixed (albeit unknown to the planner) threshold level. Such endogenous uncertainty stems from our ignorance concerning the occurrence conditions rather than from the intrinsic stochastic nature of the events. The corresponding occurrence hazard depends on the complete pollution history as well as on the current trend, giving rise to equilibrium intervals rather than the isolated equilibrium points characterizing the optimal policies under exogenous uncertainty. Moreover, T&Z (1994, 1995, 1996a) have shown that endogenous uncertainty always implies more conservation.

These apparently conflicting observations have motivated the present effort, offering a general framework for analyzing exogenous uncertainty, which accommodates C&R's irreversible events as a special case but accounts also for reversible situations. The present analysis elucidates the relation between the nature of the events (reversible vs. irreversible, exogenous vs. endogenous) and the ensuing policies (more or less conservation). It is found that both properties are indeed crucial, and reversible events always imply more conservation.

Our analysis is based on a method, denoted the $h\delta$ -method, to identify the set of possible optimal equilibria. First employed in the analysis of endogenous events (T&Z, 1995), the $h\delta$ -method shows its full power in the context of the present case of exogenous events, permitting one to characterize the equilibrium structure of the process without the need to derive the full dynamic solutions. A detailed account of the results presented here can be found in T&Z (1996b).

2. The problem

The general setup follows closely that of C&R. The state variable P represents the total pollution level. Natural purification and decay processes reduce the amount of pollution at the rate $R(P)$ which is increasing and concave in P , and $R'(P) < \infty$. The natural pollution level at which $R(P)$ vanishes is normalized at zero, i.e., $R(0) = 0$.

Anthropogenic activity increases the amount of pollution P at the emission rate g . Thus

$$dP_t / dt \equiv \dot{P} = g_t - R(P_t) \quad (2.1)$$

The pollution flow g is a by-product of production processes that generate goods. The tradeoff between the utility provided by these goods and the disutility caused by pollution is represented by the instantaneous net welfare function $B(g, P)$. This function accounts for the benefits resulting from the production activity corresponding to the emission rate g as well as for the direct environmental costs associated with the pollution level P . We assume: $\partial B / \partial P \equiv B_2(g, P) \leq 0$;

$$\partial^2 B / \partial g^2 \equiv B_{11}(g, P) < 0; \partial^2 B / \partial g \partial P \equiv B_{12}(g, P) \leq 0; \quad \text{and}$$

$$\partial^2 B / \partial P^2 \equiv B_{22}(g, P) \leq 0.$$

We also postulate the existence of a limiting pollution level \bar{P} above which the whole environmental system is bound to collapse. Thus,

$$B(g, P) \rightarrow -\infty \text{ as } P \rightarrow \bar{P}.$$

In addition to the direct costs, the planner must consider also the consequences of a possible occurrence, at any pollution level P , of an environmental event (possibly a catastrophe). The event occurrence is determined by stochastic exogenous conditions and its probability is described in terms of a state-dependent hazard-rate function $\lambda(P)$, yielding

$$\Pr\{T > t\} = 1 - F(t) = \exp\left\{-\int_0^t \lambda(P_\tau) d\tau\right\} \text{ for the distribution of the}$$

occurrence time T . We assume that $\lambda(P)$ is non-decreasing, with $\lambda'(P) < \infty$ in $[0, \bar{P}]$.

A pollution policy (or plan) consists of the emission process g_t and the associated state process P_t , $t \geq 0$. A plan is feasible if, for all t , (2.1) is satisfied, g_t is piecewise continuous and nonnegative, and $P_t \leq \bar{P}$.

Let $\varphi(P)$ be the post-event value function, representing the value that can be derived *after* the event occurred at the state P . Explicit expressions for $\varphi(P)$, depending on the nature of the events, are derived in the following section.

Taking expectation over the occurrence time T , the expected, pre-event value at any pre-event state P is expressed as

$$V(P) = \text{Max}_{\{g_t\}} \int_0^\infty \lambda(P_t) \exp \left\{ - \int_0^T \lambda(P_t) dt \right\} \left(\int_0^T B(g_t, P_t) e^{-\rho t} dt + e^{-\rho T} \varphi(P_T) \right) dT \quad (2.2)$$

subject to (2.1), $g_t \geq 0; P_t \leq \bar{P}$ and $P_0 = P$, where ρ is the time rate of discount. C&R have shown how the optimal plan associated with (2.2) with $\varphi = 0$ (the irreversible case) can be characterized via optimal control techniques. Here we follow a different approach.

First we observe that problem (2.2) is autonomous in nature, hence the *optimal* state process associated with $V(P)$ must evolve monotonically in time (T&Z, 1994). Being monotonic and bounded, the optimal pre-event state process associated with $V(P)$ must approach a steady state. The location of the possible equilibria is therefore of prime importance and we identify such states via:

The $h\delta$ -Method: This method determines whether a state level P can be an optimal steady state by comparing the steady-state policy (which maintains the emission rate g equals to the removal rate $R(P)$ until a catastrophe occurs) to an infinitesimal variation from it. If the variation improves the value, P is excluded from the list of possible steady states.

Without occurrence risk, the non-event steady state benefit is given by

$W_n(P) = B(R(P), P) / \rho$. Under uncertainty, the corresponding benefit is

$$W_e(P) = E_T \left\{ \int_0^T B(R(P), P) e^{-\rho t} dt + e^{-\rho T} \varphi(P) \right\} =$$

$$W_n(P) - [W_n(P) - \varphi(P)] E_T \{ e^{-\rho T} \}$$

where E_T denotes expectation with respect to the distribution of T . Thus

$$W_e(P) = W_n(P) - [W_n(P) - \varphi(P)]\lambda(P)/[\rho + \lambda(P)]. \quad (2.3)$$

For arbitrarily small constants $h > 0$ and δ , define the feasible variation on the steady state plan by

$$g_t^{h\delta} = \begin{cases} R(P) + \delta, & 0 \leq t < h \\ R(P_h), & t \geq h \end{cases}. \quad (2.4)$$

With this plan, we find for all $t \leq h$, using the fact that $R'(P)$ is bounded,

$$P_t - P = \int_0^t [R(P) + \delta - R(P_s)]ds = t\delta + o(t\delta). \quad (2.5)$$

The expected benefit associated with $g_t^{h\delta}$, denoted $V^{h\delta}(P)$, is evaluated from

(2.2), neglecting terms that are $o(h\delta)$ as in (2.5). One finds

$$V^{h\delta}(P) - W_e(P) = L_e(P)h\delta / \rho + o(h\delta) \quad (2.6)$$

where $L_e(P) = \rho[B_1(R(P), P) + W_e'(P)]$ is denoted as the *evolution function* of the optimization problem. The crucial relation between the evolution function and the equilibrium structure is explained below.

Without occurrence risk $\lambda(P)$ vanishes, $W_e(P) = W_n(P)$ and one obtains

$$L(P) = B_1(R(P), P)[\rho + R'(P)] + B_2(R(P), P) \quad (2.7)$$

for the non-event evolution function (T&Z, 1994). Under uncertainty, we

introduce the reduced hazard rate $\lambda_e(P) = \rho\lambda(P)/[\rho + \lambda(P)]$ and express the uncertainty evolution function $L(P)$ in the form

$$L_e(P) = L(P) - \{[W_n(P) - \varphi(P)]\lambda_e(P)\}'. \quad (2.8)$$

Equation (2.6) determines the equilibrium structure of the optimization problem (2.2). When $L_e(P) \neq 0$, the state level P cannot qualify as a steady state because one can choose the constants h and δ small enough, with δ and $L_e(P)$ having the same sign, to ensure that $V^{h\delta}(P) - W_e(P) > 0$, so that the steady state policy yielding $W_e(P)$ is sub-optimal. Thus, only the roots of $L_e(P)$ can be equilibrium states. The only possible exception is the end-point level $P = 0$, since this unpolluted level can be an equilibrium state if $L_e(0) < 0$, because $\delta < 0$ is not feasible for this state. (The upper level \bar{P} could also be an equilibrium state if $L_e(\bar{P}) \geq 0$, because $\delta > 0$ is not feasible for this overly polluted state. However, the assumed properties of $B(g, \bar{P})$ ensure that $L_e(\bar{P})$ must be negative.)

Returning to the non-event problem, we note that the assumed properties of B and R ensure that $L(P)/[\rho + R'(P)]$ is strictly decreasing, while $[\rho + R'(P)]$ is positive. Since $L(\bar{P})$ must be negative, there exists a unique state level \hat{P} in $[0, \bar{P})$, satisfying

$$\begin{cases} \hat{P} = 0 & \text{if } L(0) \leq 0 \\ L(\hat{P}) = 0 & \text{otherwise} \end{cases} \quad (2.9)$$

Let P_t^n denote the optimal state process of the non-event problem. As

P_t^n progresses monotonically in time and is bounded between 0 and \bar{P} , it must

converge to a steady state. The $h\delta$ -method, then, implies that \hat{P} is the unique equilibrium level to which P_t^n converges from any initial level.

3. Catastrophic events

In this section we consider three types of events whose differences are manifested by their corresponding post-event value functions. Our major concern is the question of conservation: under which conditions does the presence of event uncertainty imply less pollution.

3.1 Irreversible events

Irreversible events have been studied by C&R to model catastrophes that reduce social welfare to zero. For these events $\varphi(P) \equiv 0$ and (2.8) reduces to

$$L_e(P) = L(P) - [W_n(P)\lambda_e(P)]' \quad \text{or}$$

$$L_e(P) = \left\{ L(P) + \lambda(P)B_1(R(P), P) - \frac{\lambda'(P)B(R(P), P)}{\rho + \lambda(P)} \right\} \frac{\rho}{\rho + \lambda(P)} \quad (3.1)$$

It is easily verified that the roots of $L_e(P)$ satisfy eq. (46) of C&R which

identifies the steady states. Below \hat{P} , when $L(P) > 0$, one finds that

$$B_1(R(P), P) > -B_2(R(P), P)/[\rho + R'(P)] \geq 0. \quad \text{Therefore, if the hazard rate}$$

is constant, $L_e(P) = [L(P) + \lambda B_1(R(P), P)]\rho/[\rho + \lambda] > 0$ and L_e can have

no roots at $P < \hat{P}$. This is consistent with C&R's observation that a constant

hazard rate implies *more* pollution at the steady state than the equilibrium

pollution of the non-event situation. However, if $\lambda(P)$ increases strongly with P ,

the term involving $\lambda'(P)$ may dominate the sign of L_e , yielding a root below

\hat{P} . In this case, a more conservative plan will follow. A detailed interpretation of these results is provided by these authors.

3.2 *Single-occurrence reversible events*

A catastrophe of this type can occur only once, inflicting a penalty but not terminating the economic activity. For example, the extinction of a certain species results in a biodiversity loss (penalty) but otherwise does not cease the process that caused extinction (see T&Z, 1994). Of course, species extinction is an irreversible phenomenon, but its effect on the planning problem is recoverable, explaining our use of the term "reversible" to describe these events in our context. For such events, the optimal post-event policy is to follow the non-event policy, hence the post-event value assumes the form $\varphi(P) = V_n(P) - \psi(P)$, where

$V_n(P) = \text{Max}_{\{g_t\}} \int_0^{\infty} B(g_t, P_t) e^{-\rho t} dt$ subject to (2.1) is the non-event value and

$\psi(P)$ is the penalty associated with a catastrophe at the level P . We assume that the penalty $\psi(P)$ is differentiable and non-decreasing. The evolution function L_e of (2.8) specializes to

$$L_e(P) = L(P) - \{[W_n(P) - V_n(P)]\lambda_e(P)\}' - \{\psi(P)\lambda_e(P)\}'. \quad (3.2)$$

It is seen that the pollution level P plays a dual role: it affects the occurrence probability (through the hazard rate $\lambda(P)$), and it determines its consequences (through the penalty function $\psi(P)$). We show now that single-occurrence reversible events induce *more* conservation. Equivalently

Proposition 1: For single-occurrence reversible events, the optimal equilibrium state cannot fall above \hat{P} .

Proof: The idea is to show, for any state $P > \hat{P}$, that the non-event plan yields a higher value than the steady state policy. By the "non-event" plan we mean the plan that follows the optimal process P_t^n associated with the non-event problem both before and after occurrence, paying the penalty when required. The "steady state" policy means setting $g = R(P)$ until occurrence and following the non-event plan thereafter.

The benefit generated by the non-event plan is

$$U(P) = V_n(P) - E_T \{ e^{-\rho T} \psi(P_T^n) \} = \\ V_n(P) - \int_0^\infty \psi(P_T^n) \lambda(P_T^n) \exp \left\{ - \int_0^T [\lambda(P_t^n) + \rho] dt \right\} dT$$

where P_T^n is the level along the non-event trajectory at which the event occurs.

From (2.3), the benefit generated by the steady state policy is

$$W_e(P) = \{ B(R(P), P) + [V_n(P) - \psi(P)] \lambda(P) \} / [\rho + \lambda(P)].$$

Since $U(P)$ is feasible, for a state P to be an optimal equilibrium state we must

have $W_e(P) \geq U(P)$. However, when passing through $P > \hat{P}$, the non-event

process P_t^n decreases, hence $\lambda(P_T^n) \leq \lambda(P)$ for all T . Thus,

$$\rho \lambda(P_T^n) + \lambda(P) \lambda(P_T^n) \leq \rho \lambda(P) + \lambda(P) \lambda(P_T^n) = \lambda(P) [\rho + \lambda(P_T^n)] \text{ or,}$$

$$\lambda(P_T^n) \leq \lambda(P) [\rho + \lambda(P_T^n)] / [\rho + \lambda(P)]. \text{ Also } \psi(P_T^n) \leq \psi(P) \text{ and}$$

$$U(P) \geq V_n(P) - \psi(P) \int_0^\infty \lambda(P_T^n) \exp \left\{ - \int_0^T [\lambda(P_t^n) + \rho] dt \right\} dT$$

$$\begin{aligned}
&\geq V_n(P) - \frac{\psi(P)\lambda(P)}{\rho + \lambda(P)} \int_0^\infty [\rho + \lambda(P_t^n)] \exp\left\{-\int_0^T [\lambda(P_t^n) + \rho] dt\right\} dT \\
&= V_n(P) + \frac{\psi(P)\lambda(P)}{\rho + \lambda(P)} \exp\left\{-\int_0^T [\lambda(P_t^n) + \rho] dt\right\} \Big|_{T=0}^{T=\infty} = V_n(P) - \frac{\psi(P)\lambda(P)}{\rho + \lambda(P)}
\end{aligned}$$

Thus, $W_e(P) \geq U(P)$ implies

$$[B(R(P), P) + V_n(P)\lambda(P)]/[\rho + \lambda(P)] \geq V_n(P), \text{ or}$$

$$B(R(P), P)/\rho \geq V_n(P), \text{ which is impossible unless } P = \hat{P}.$$

Note that the sum of the first two terms on the right-hand side of (3.2) is positive below \hat{P} , while the last term is negative (T&Z, 1996b). At \hat{P} , the first two terms vanish and $L_e(\hat{P}) < 0$. The existence of a root of L_e (which can serve as a steady state) in $(0, \hat{P})$ depends on the competition between this positive sum and the negative $-\psi(P)\lambda_e(P)$ contribution. If the penalty is very large, or if it changes rapidly, such a root does not exist and the expected loss due to the event will drive the process towards the unpolluted, natural state.

3.3 Recurrent reversible events

Consider a situation in which enhanced pollution, combined with certain atmospheric conditions, can trigger the outburst of a certain disease, which can be cured at a cost. Such an unfortunate scenario can repeat over and over again, when certain conditions reoccur. Unlike the previous case of a single event, occurrence here does not entail a resolution of uncertainty nor a passage to the non-event situation. Rather, apart from the inflicted penalty, nothing new is learnt

and the ensuing pollution control problem is the same as before occurrence. For such events, therefore, $\varphi(P) = V(P) - \psi(P)$ in (2.3). However, the derivation of the evolution function L_e based on this specification does not produce a useful form due to the presence of the value function $V(P)$, which is not a-priori known.

In deriving the relevant L_e function, the $h\delta$ -method considers steady state policies and small variations from them. For recurrent events the steady state policy should remain unaltered after occurrence, and the appropriate post-event value to be used in (2.3) is $\varphi(P) = W_e(P) - \psi(P)$, where $W_e(P)$ is the value under the steady state policy, yielding

$$W_e(P) = W_n(P) - \lambda(P)\psi(P)/\rho. \quad (3.3)$$

Using (2.8) and (3.3) we obtain the evolution function for recurrent events

$$L_e(P) = L(P) - [\psi(P)\lambda(P)]'. \quad (3.4)$$

As both the penalty and the hazard functions are positive and non-decreasing,

$[\psi(P)\lambda(P)]' \geq 0$ and $L_e(P)$ can have no roots above \hat{P} . Therefore, recurrent events induce conservation, in similarity with single-occurrence reversible events.

The existence of a root below \hat{P} depends on the magnitude of $L(P)$ vis-a-vis the contribution of the $[\psi(P)\lambda(P)]'$ penalty term. Indeed, if the product

$\psi(P)\lambda(P)$ is convex in P , this root is unique. We have thus established

Proposition 2: For recurrent events, $P > \hat{P}$ cannot be a steady state.

When $\psi(P)\lambda(P)$ is convex in P , there exists a unique equilibrium level, approached monotonically by the state process from any initial level.

It is remarkable that the $h\delta$ -method can locate the steady state and characterize the dynamics of the optimal state process without actually solving for it.

4. Concluding comments

This work extends C&R's analysis of irreversible events to two types of reversible events: single-occurrence and multiple-occurrence (recurrent) events. While it is possible that the threat of irreversible events induces more pollution, we show here that reversible events, under quite a general condition, induce more conservation (less pollution). The "general condition" is that both the hazard rate of the occurrence date and the penalty the event inflicts are non-decreasing functions of the pollution level.

Why are the irreversible events so different? Formally, one could view them as reversible events, with a penalty that just equals the value forgone due to occurrence. The value function, however, generally decreases with pollution whereas for reversible events a non-decreasing penalty function has been postulated. For the latter type of events, pollution increases both the hazard rate and the penalty and both effects lead to more conservation, whereas for irreversible events pollution decreases the "penalty" and the conflicting trends may lead to less conservation. In this respect, the exogenous events considered here vary greatly relative to the endogenous events, for which event uncertainty always entails more conservation.

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